

The charged black hole/string transition

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ABSTRACT: We generalize the discussion of [1] to charged black holes. For the two dimensional charged black hole, which is described by an exactly solvable worldsheet theory, a transition from the black hole to the string phase occurs when the Hawking temperature of the black hole reaches a limiting value, the temperature of free strings with the same mass and charge. At this point a tachyon winding around euclidean time in the euclidean black hole geometry, which has a non-zero condensate, becomes massless at infinity, and the horizon of the black hole is infinitely smeared. For Reissner-Nordström black holes in $d \geq 4$ dimensions, the exact worldsheet CFT is not known, but we propose that it has similar properties. We check that the leading order solution is in good agreement with this proposal, and discuss the expected form of α' corrections.

KEYWORDS: Black Holes in String Theory, Conformal Field Models in String Theory, Superstrings and Heterotic Strings.

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1. Introduction

In [1] a new picture of the string/black hole transition (see [2–5] for earlier discussions) was proposed. According to this picture, as the Hawking temperature of a black hole T_{bh} increases, the geometry develops large fluctuations in a wider and wider region around the horizon. When T_{bh} approaches the temperature of fundamental strings with the same quantum numbers as the black hole, T_f , the size of this stretched horizon goes to infinity and the black hole becomes indistinguishable from a gas of fundamental strings.

The divergence of the size of the stretched horizon can be seen by studying the euclidean black hole geometry. It was argued in [1] that in this geometry there is a non-zero condensate of the closed string tachyon wound around the euclidean time direction. This is known to be the case for two dimensional black holes related to $SL(2, \mathbb{R})$ CFT, and was conjectured in [1] to be the case for higher dimensional black holes as well. The mass of the wound tachyon at infinity is large for black holes whose Hawking temperature is much lower than the corresponding fundamental string temperature, but it goes to zero when the two temperatures coincide. In this case, the tachyon condensate smears the euclidean black hole geometry all the way to infinity. It was argued in [1] that this is the euclidean manifestation of the infinite size of the stretched horizon for the case $T_{bh} = T_f$.

In [1] the above picture was explored and tested for the case of uncharged black holes in two dimensional spacetime with a linear dilaton, and for Schwarzschild black holes in $d \geq 4$ dimensions. The purpose of this note is to generalize the discussion to charged black holes. There are a number of reasons to study this generalization. One is that it is known [4] that the string/black hole transition for charged black holes can occur at temperatures that are arbitrarily low compared to the Hagedorn temperature. Thus, it is natural to ask whether and how the mass of the wound tachyon at infinity can go to zero at such temperatures. In particular, in the extremal limit in which the mass and charge coincide, both T_{bh} and

T_f go to zero. A natural question is what happens then? Also, Wick rotating a charged black hole to euclidean space leads to a background with an imaginary electric field, unless we Wick rotate the charge as well. In discussing the normalizability of states it seems important to consider a real euclidean background, i.e. to Wick rotate the charge, but then it is not clear how to relate the results to the original Minkowski problem.

The particular cases we will study are the two dimensional black hole of [6], and Reissner-Nordström (RN) black holes in $d \geq 4$ dimensions. The former is an exact (in α') solution of string theory [7–9]. In section 2 we show that the euclidean solution involves a condensate of a tachyon which winds once around euclidean time. This condensate becomes non-normalizable precisely at the point where the Hawking temperature of the black hole reaches the (limiting) fundamental string temperature. At the transition, the fundamental string and black hole entropies coincide [5].

In section 3, we study RN black holes in $d \geq 4$ dimensions. Like in the Schwarzschild case, the exact CFT corresponding to these geometries in string theory is not known, but as in [1], we can test the picture by using the leading order solution. We find that at the transition, when $T_{bh} = T_f$, the entropy computed from the leading order black hole solution differs from the fundamental string one by a factor $(d-3)/(d-2)$, independently of the charge. Like in [1], we attribute this disagreement to α' corrections. In section 4 we discuss some aspects of our results.

2. Charged two dimensional black hole

2.1 Lorentzian black hole

The two dimensional charged black hole of [6] is described by the line element

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{Q^2 r^2 f(r)}, \tag{2.1}$$

dilaton

$$\Phi(r) = -\frac{1}{2} \ln(r/Q), \tag{2.2}$$

and U(1) gauge field

$$A_t(r) = \frac{q}{r}. \tag{2.3}$$

The function $f(r)$ in (2.1) is related to the mass m and charge q of the black hole as follows

$$f(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2} = \left(1 - \frac{r_-}{r}\right) \left(1 - \frac{r_+}{r}\right). \tag{2.4}$$

The second equality in (2.4) defines the inner (r_-) and outer (r_+) horizons of the black hole, which are given by

$$r_{\pm} = m \pm \sqrt{m^2 - q^2}. \tag{2.5}$$

At large r , $f(r) \rightarrow 1$ and the geometry approaches a linear dilaton one,

$$\begin{aligned} ds^2 &= d\phi^2 - dt^2, \\ \Phi &= -\frac{Q}{2}\phi. \end{aligned} \tag{2.6}$$

Here and below we set $\alpha' = 2$, such that the central charge of ϕ is $c_\phi = 1 + 3Q^2$.

The solution (2.1) – (2.3) is very reminiscent of the four dimensional Reissner-Nordström black hole, and can be used as a toy model for studying charged black holes in higher dimensions. It has the advantage that string propagation in this geometry is described by a coset model [8], and is (classically) exactly solvable. The coset in question,

$$\frac{\text{SL}(2, \mathbb{R})_k \times \text{U}(1)_L}{\text{U}(1)}, \tag{2.7}$$

can be constructed as follows (see [10] for a recent discussion). We start with the supersymmetric $\text{SL}(2, \mathbb{R})$ WZW model at level k , whose central charge is $c = \bar{c} = 3 + \frac{6}{k} + \frac{3}{2}$, and add to it a left-moving supersymmetric $\text{U}(1)$ current with $c = \frac{3}{2}$, $\bar{c} = 0$. To get the black hole (2.1) – (2.3) we gauge a $\text{U}(1)$ current whose right-moving component is one of the spacelike $\text{U}(1)$'s in $\text{SL}(2, \mathbb{R})_k$, while the left-moving component is a linear combination of a spacelike $\text{U}(1)$ in $\text{SL}(2, \mathbb{R})$ and the $\text{U}(1)_L$ in (2.7). The free parameter that determines this linear combination corresponds to the charge to mass ratio of the black hole. When the left-moving component of the gauged $\text{U}(1)$ lies entirely in $\text{SL}(2, \mathbb{R})$, the coset describes the uncharged black hole ($q = 0$). When it is entirely in $\text{U}(1)_L$ (2.7), one gets the extremal black hole, with $q = m$.

In the coset description, the maximal extension of the background (2.1) – (2.3) splits into different regions, each of which is described by its own set of natural coordinates. The metric, dilaton and gauge field in the region outside the outer horizon are given by

$$ds^2 = d\phi^2 - \left(\frac{\tanh \frac{Q}{2}\phi}{1 - a^2 \tanh^2 \frac{Q}{2}\phi} \right)^2 d\theta^2, \tag{2.8}$$

$$\Phi(\phi) = \Phi_0 - \frac{1}{2} \log \left(1 + (1 - a^2) \sinh^2 \frac{Q}{2}\phi \right), \tag{2.9}$$

and

$$A_\theta(\phi) = \frac{a \tanh^2 \frac{Q}{2}\phi}{1 - a^2 \tanh^2 \frac{Q}{2}\phi}, \tag{2.10}$$

where a is a function of the charge to mass ratio of the black hole,

$$a^2 = \frac{r_-}{r_+}. \tag{2.11}$$

In particular, it varies in the range $0 \leq |a| \leq 1$. $a = 0$ corresponds to the uncharged black hole, which was studied from the current perspective in [1], while $|a| = 1$ corresponds to the extremal case, $|q| = m$. The sign of a can be changed by taking the gauge field to minus itself, i.e. by flipping the sign of all charges. Below we will take a and q to be positive.

The coordinates r in (2.1)–(2.3) and ϕ in (2.8)–(2.10) are related by a coordinate transformation that can be read off (2.2), (2.9), while θ and t are related by

$$t = -\frac{\theta}{1 - a^2}. \tag{2.12}$$

The gauge field (2.10) approaches a non-zero value at infinity,

$$A_\theta(\phi \rightarrow \infty) \rightarrow \frac{a}{1 - a^2} . \tag{2.13}$$

This is different from (2.3) which goes to zero for large r . The discrepancy can be eliminated by adding the constant $-a$ to the right hand side of (2.3). Adding a constant to the electric potential does not change the classical dynamics in the black hole background. Nevertheless, we will see below that the presence of the non-zero asymptotic gauge field at infinity is important.

The coordinate θ is better suited than t for studying the extremal limit, in which (2.11) $a^2 \rightarrow 1$ and the rescaling factor in (2.12) diverges. In this limit, the qualitative structure of the background (2.8)–(2.10) changes. Instead of the asymptotically linear dilaton behavior (2.6), it approaches AdS_2 with constant dilaton,

$$\begin{aligned} ds^2 &= d\phi^2 - \frac{1}{4} \sinh^2(Q\phi) d\theta^2 , \\ \Phi &= \Phi_0 , \\ A_\theta(\phi) &= \sinh^2 \frac{Q}{2} \phi . \end{aligned} \tag{2.14}$$

We will comment further on this limit in section 4.

2.2 Euclidean black hole and thermodynamics

We next turn to the euclidean continuation of the black hole solution, which is obtained by taking $\theta \rightarrow i\theta$ in (2.8). In order to keep the gauge field (2.10) real, we also need to take $a \rightarrow -ia$. Looking back at (2.11), this can be thought of as taking $r_- \rightarrow -r_-$, keeping r_+ fixed.

The resulting solution is the euclidean $\frac{SL(2, \mathbb{R}) \times U(1)_L}{U(1)}$ coset model, recently studied in [11]. The background fields are

$$ds^2 = d\phi^2 + R^2(\phi) dy^2 , \tag{2.15}$$

$$\Phi(\phi) = \Phi_0 - \frac{1}{2} \log \left(1 + (1 + a^2) \sinh^2 \frac{Q}{2} \phi \right) , \tag{2.16}$$

$$A_y(\phi) = aR(\phi) \tanh \frac{Q}{2} \phi , \tag{2.17}$$

where

$$R(\phi) = \frac{2}{Q} \frac{\tanh \frac{Q}{2} \phi}{1 + a^2 \tanh^2 \frac{Q}{2} \phi} . \tag{2.18}$$

Regularity of the geometry at the tip, $\phi = 0$, requires $y \equiv Q\theta$ to be periodic,

$$y \sim y + 2\pi . \tag{2.19}$$

Near the boundary at $\phi = \infty$, the background (2.15)–(2.18) approaches

$$\mathbb{R}_\phi \times S^1 \times U(1)_L . \tag{2.20}$$

The dilaton depends linearly on ϕ , as in (2.6). The radius of S^1 is

$$R_\infty = \frac{2}{Q(1+a^2)} . \tag{2.21}$$

The gauge field approaches

$$A_y = aR_\infty , \tag{2.22}$$

which gives rise to a non-zero Polyakov-Wilson loop

$$e^{i \int_0^{2\pi} dy A_y} = e^{i2\pi R_\infty a} . \tag{2.23}$$

In the coset description, a is fixed by the charge to mass ratio of the black hole via (2.11). One can understand its value directly in gravity as follows. In the coordinates (r, t) , (2.1), the gauge field (2.17) is given by

$$A_t(r) = \frac{q}{r} - a . \tag{2.24}$$

The choice (2.11) has the property that A_t vanishes at the horizon, $r = r_+$, which is a necessary condition to avoid a singularity there.

From the point of view of black hole thermodynamics, the euclidean charged black hole (2.15)–(2.18) contributes to the canonical partition sum

$$Z(\beta, a) = \text{Tr} e^{-\beta(H+iaQ)} , \tag{2.25}$$

where the inverse black hole temperature $\beta = \beta_{bh}$ and chemical potential a are determined by the linear dilaton slope Q and charge to mass ratio q/m :

$$\begin{aligned} (1-a^2) \frac{\beta_{bh}}{4\pi} &= \frac{1}{Q} , \\ a &= \tan \frac{\alpha}{2} . \end{aligned} \tag{2.26}$$

α parametrizes the charge to mass ratio of the black hole,

$$\sin \alpha = \frac{q}{m} . \tag{2.27}$$

Upon Wick rotation to Minkowski space, one finds the black hole (2.1)–(2.3) in thermal equilibrium with a heat bath with the Boltzmann factor

$$e^{-\beta(H-aQ)} . \tag{2.28}$$

The black hole entropy corresponding to (2.26) can be obtained by using the thermodynamic relations

$$\begin{aligned} \beta &= \left(\frac{\partial S}{\partial m} \right)_q , \\ -\beta a &= \left(\frac{\partial S}{\partial q} \right)_m . \end{aligned} \tag{2.29}$$

It is given by

$$S_{bh}(m, q) = \frac{2\pi}{Q} \left(m + \sqrt{m^2 - q^2} \right) . \quad (2.30)$$

The partition sum associated with (2.28) is

$$\text{Tre}^{-\beta(H-aQ)} \sim \int dm e^{S_{bh}(m, q) - \beta m(1-a \sin \alpha)} . \quad (2.31)$$

In principle we have to sum over all α 's (2.27), but we expect the sum to be dominated by states with a particular value of this parameter. Indeed, solving for m in terms of the entropy and substituting into (2.31), we find that the integrand in (2.31) goes like

$$e^{(1 - \frac{Q\beta}{2\pi} f(\alpha)) S_{bh}} , \quad (2.32)$$

where

$$f(\alpha) = \frac{1 - a \sin \alpha}{1 + \cos \alpha} . \quad (2.33)$$

For large entropy, the partition sum (2.31) is dominated by states corresponding to a minimum of $f(\alpha)$. One can check that this minimum lies at $a = \tan \frac{\alpha}{2}$, which is precisely the value given in (2.26). This provides another way of understanding why a black hole with a given charge to mass ratio contributes to the partition sum (2.31) only for a particular value of the chemical potential.

The value of $f(\alpha)$ (2.33) at the minimum is

$$f(\alpha)|_{\min} = \frac{1}{2}(1 - a^2) . \quad (2.34)$$

Plugging (2.34) and the black hole temperature (2.26) into (2.32) we see that the free energy vanishes, in agreement with the fact that the density of states with fixed α , (2.30), exhibits Hagedorn growth.

An important property of the euclidean black hole background is that in addition to the fields (2.15)–(2.18) it has a condensate of a closed string tachyon winding around the euclidean time circle. For the uncharged black hole (which corresponds to $a = 0$) this is a consequence of the generalization of the FZZ correspondence between the cigar and Sine-Liouville theories [12, 13] to the fermionic string [14]. The generalization to $a \neq 0$ can be obtained by performing a rotation on the left-movers (by the angle α (2.27)), which mixes the $SL(2, \mathbb{R})$ and $U(1)_L$ factors.

At large ϕ the geometry (2.20)–(2.22) is flat, and the vertex operator of the winding tachyon has the form

$$T \sim e^{\tilde{\beta}\phi + ip_L \cdot x_L + ip_R \cdot x_R} . \quad (2.35)$$

The left/right moving momentum vector $(p_L; p_R)$ is determined by the radius of the S^1 , (2.21), and Wilson line (2.22) (see e.g. eq. (11.6.17) in [15]). For a state with winding one around the circle one finds

$$(p_L; p_R) = \frac{1}{2} R_\infty (1 - a^2, 2a; -(1 + a^2)) = \frac{1}{Q} (\cos \alpha, \sin \alpha; -1) . \quad (2.36)$$

In the second equality we used the value of R_∞ (2.21) and the relation between a and α (2.26). The first component of p_L in (2.36) is the left-moving momentum along S^1 , while the second component is in the $U(1)_L$ direction in (2.20). The last component of (2.36) is the right-moving momentum on S^1 . The length of the left and right moving momentum vectors is

$$p_L^2 = p_R^2 = \frac{1}{4} R_\infty^2 (1 + a^2)^2 = \frac{1}{Q^2}, \quad (2.37)$$

independent of a . The value of $\tilde{\beta}$ in (2.35) is the same as in the uncharged case [14], $\tilde{\beta} = -1/Q$.

The fact that the tachyon (2.35) has a non-zero expectation value in the euclidean coset implies that the worldsheet lagrangian contains the $N = 2$ Liouville perturbation

$$\delta S = \mu \int d^2 z d^2 \theta e^{-\frac{1}{Q}(\phi + i(\cos \alpha, \sin \alpha) \cdot x_L - i x_R)} + \text{c.c.} . \quad (2.38)$$

For small Q , the perturbation (2.38) provides a small correction to the euclidean background (2.15)–(2.18) since it goes rapidly to zero at large ϕ . As Q increases, it becomes more and more important and eventually takes over. For $Q^2 > 2$, it becomes non-normalizable [16], and the euclidean black hole ceases to contribute to the partition sum (2.25); see [17, 5] for recent discussions.

From the spacetime point of view, the asymptotic mass of the winding tachyon is a sum of three contributions: the mass of the closed string tachyon (which is -1 in our units), a contribution due to winding (2.37), and a factor $Q^2/4$ due to the linear dilaton (2.6),

$$m_\infty^2 = -1 + \frac{1}{Q^2} + \frac{Q^2}{4} = \left(\frac{1}{Q} - \frac{Q}{2}\right)^2 . \quad (2.39)$$

For small Q , the tachyon is very massive, and its wave function goes rapidly to zero (2.38). As Q increases, the tachyon becomes lighter, and modifies the geometry in a larger region in ϕ . For $Q^2 \rightarrow 2$, the size of this region diverges. For larger Q , the tachyon becomes massive again, but it does not have a normalizable state in the throat, and the background (2.38) remains non-normalizable.

2.3 The black hole/string transition

As is familiar from other contexts in string theory (see e.g. [18]), the presence of a light tachyon wound around euclidean time for $Q^2 \simeq 2$ encodes the effects of highly excited fundamental strings. The transition at $Q^2 = 2$ is from a black hole phase to a perturbative string one [5]. In this subsection we describe the high energy thermodynamics of charged perturbative strings, and the transition between the black hole and string phases.

The entropy of perturbative single string states with mass m and charge q under $U(1)_L$ in (2.20) is given by

$$S_f(m, q) = 2\pi \sqrt{1 - \frac{Q^2}{4}} \left(m + \sqrt{m^2 - q^2}\right) . \quad (2.40)$$

The corresponding temperature is $T_f = 1/\beta_f$ (2.29), with

$$(1 - a^2) \frac{\beta_f}{4\pi} = \sqrt{1 - \frac{Q^2}{4}}. \quad (2.41)$$

The chemical potential a (2.29) is the same as in the black hole case (2.26). This is due to the fact that the fundamental string entropy (2.40), like the black hole entropy (2.30), is a function of the combination $m + \sqrt{m^2 - q^2}$. It is also interesting that the chemical potential a only depends on $\frac{q}{m}$ and not on Q , i.e. it does not receive α' corrections. We will see that something similar seems to happen for d -dimensional RN black holes.

The temperature T_f is in fact a limiting temperature for strings with chemical potential a . To see that, consider the free string partition sum (2.31),

$$\text{Tr} e^{-\beta(H-aQ)} \sim \int dm e^{S_f(m,q) - \beta m(1-a \sin \alpha)}, \quad (2.42)$$

where α is defined as in (2.27) again. Performing the sum (2.42) over states with a given q/m , or given α , one finds that the integral diverges for $\beta < \beta_c(\alpha)$, where

$$\beta_c(\alpha) = \frac{2\pi}{f(\alpha)} \sqrt{1 - \frac{Q^2}{4}}, \quad (2.43)$$

and $f(\alpha)$ is given by (2.33). The states which lead to the smallest critical temperature are those for which $f(\alpha)$ is smallest. The value of α at the minimum was found in the previous subsection to be given by the second line of (2.26). Plugging in the value of $f(\alpha)$ at the minimum, (2.34), we conclude that the partition sum (2.42) diverges for $\beta < \beta_f$ (2.41). Thus, $1/\beta_f$ is a limiting temperature in free string theory with the chemical potential a (2.26).¹

Comparing (2.26) and (2.41) we see that

$$\frac{T_{bh}}{T_f} = Q \sqrt{1 - \frac{Q^2}{4}}. \quad (2.44)$$

For small Q the black hole temperature T_{bh} is much smaller than the limiting temperature T_f . As Q increases, the ratio (2.44) increases until it goes to one at $Q^2 = 2$. At that point the black hole temperature approaches the limiting one and the black hole becomes indistinguishable from a gas of fundamental strings at the limiting temperature. Therefore, it must be that the black hole entropy approaches the fundamental string one [1]. This is indeed the case [5], as can be checked by comparing (2.30) and (2.40).

To understand the transition better it is useful to compute the size of the stretched horizon of the black hole. As in [3, 1], the stretched horizon can be defined as the region in which the locally measured Hawking temperature exceeds the limiting temperature T_f . The local Hawking temperature is the temperature at infinity divided by the red-shift factor $\sqrt{g_{00}} = \sqrt{f(r)}$ (see (2.1)). In the coordinates (2.8), it takes the form

$$T_{bh}(\phi) = \frac{Q}{4\pi} \frac{1 - a^2 \tanh^2 \frac{Q}{2} \phi}{\tanh \frac{Q}{2} \phi}. \quad (2.45)$$

¹For the uncharged case $\alpha = a = 0$, it reduces to the Hagedorn temperature in the linear dilaton throat.

The stretched horizon is the region in which $T_{bh}(\phi) > T_f$, or

$$\frac{Q}{4\pi} \frac{1 - a^2 \tanh^2 \frac{Q}{2}\phi}{\tanh \frac{Q}{2}\phi} > \frac{1}{2\pi\sqrt{4 - Q^2}} (1 - a^2) . \quad (2.46)$$

For small Q , the size of this region is $\delta\phi \simeq 2/(1 - a^2)$. As Q increases, its size grows, until it diverges for $Q = \sqrt{2}$.

It is interesting to note that the mass of the wound tachyon, (2.39), is related to the black hole and fundamental string temperatures (2.26), (2.41) by the relation

$$m_\infty^2 = (1 - a^2)^2 \left[\left(\frac{\beta_{bh}}{4\pi} \right)^2 - \left(\frac{\beta_f}{4\pi} \right)^2 \right] . \quad (2.47)$$

This relation is very reminiscent of the one satisfied by the thermal scalar in critical string theory at finite temperature [18]. It generalizes the one that was obtained in [1] for the uncharged case $a = 0$, and we propose the same interpretation as the one given there. In general, the thermal scalar encodes properties of highly excited strings, and the fact that it has a non-zero condensate (2.38) suggests that these strings are excited in the black hole background.

3. Reissner-Nordström black holes

The d -dimensional RN black hole is a solution of the equations of motion of Einstein gravity coupled to a U(1) gauge field. Unlike the two dimensional case, this solution is expected to receive α' corrections, which we will briefly comment on below.

The line element is given by [19]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2 , \quad (3.1)$$

with

$$f(r) = \left(1 - \frac{r_-^{d-3}}{r^{d-3}} \right) \left(1 - \frac{r_+^{d-3}}{r^{d-3}} \right) . \quad (3.2)$$

There is also a gauge potential $A_t \sim q/r^{d-3}$. In comparing to string theory we will take A_μ to be a Kaluza-Klein gauge field, whose charge is the left-moving momentum in a compact direction. In particular, the solutions we will study have vanishing RR fields.

The inner and outer horizons r_\pm in (3.2) are given in terms of the mass and charge by

$$r_\pm^{d-3} = \frac{8\pi G_d}{(d-2)\Omega_{d-2}} \left(m \pm \sqrt{m^2 - q^2} \right) . \quad (3.3)$$

Ω_{d-2} is the area of the unit $(d-2)$ -sphere, $\Omega_{d-2} = 2\pi^{\frac{d-1}{2}}/\Gamma(\frac{d-1}{2})$, and G_d is the d -dimensional Newton constant. The charge q is normalized such that the extremal case corresponds to $q = m$.

The Bekenstein-Hawking entropy of the black hole (3.1) is given by the area formula,

$$S_{bh} = \frac{\Omega_{d-2}}{4G_d} r_+^{d-2} . \quad (3.4)$$

From it we can read off the temperature and chemical potential using (2.29). One finds

$$\begin{aligned}
 a &= \left(\frac{r_-}{r_+} \right)^{\frac{d-3}{2}} = \tan \frac{\alpha}{2}, \\
 \beta_{bh} &= \frac{4\pi}{d-3} \frac{r_+^{d-2}}{r_+^{d-3} - r_-^{d-3}} = \frac{4\pi r_+}{(d-3)(1-a^2)},
 \end{aligned}
 \tag{3.5}$$

where α is defined in (2.27). Like in the two dimensional case, a in (3.5) can be thought of as the value of the gauge field at infinity. It plays a similar role in the euclidean black hole geometry.

Note also that as a function of q/m the chemical potential a is the same in all cases (see (2.26), (3.5)). In the two dimensional case the relation between a and q/m was explained by considering the partition sum (2.31) and asking which states make the biggest contribution to it. The same explanation can be used in the present case. Indeed, the entropy (3.4) is a function of r_+ , which can be written in terms of m and the parameter α (2.27) as follows:

$$r_+^{d-3} = \frac{8\pi G_d}{(d-2)\Omega_{d-2}} m(1 + \cos \alpha).
 \tag{3.6}$$

Repeating the logic of equations (2.31)–(2.34) one finds that the largest contribution to (2.31) for given entropy comes from states with α that is related to a via the first line of (3.5).

In the two dimensional case of section 2, we saw that the euclidean black hole CFT includes a condensate of the closed string tachyon with winding one around euclidean time (2.38). Following [1], we postulate that a similar condensate exists in the euclidean Reissner-Nordström solution, which is obtained from the lorentzian geometry (3.1) by taking $t \rightarrow it$ and $r_-^{d-3} \rightarrow -r_-^{d-3}$. The resulting geometry in the (r, t) directions is a semi-infinite cigar with asymptotic radius

$$R_\infty = \frac{r_+}{d-3} \frac{2}{1+a^2}.
 \tag{3.7}$$

The tachyon wraps the euclidean time direction, in the presence of a Wilson line a (3.5), as in section 2.

At large r the vertex operator of the winding tachyon is

$$T \sim \frac{1}{r^{d-3}} e^{-k_0 r + i p_L \cdot x_L + i p_R \cdot x_R},
 \tag{3.8}$$

where

$$(p_L; p_R) = \frac{1}{2} R_\infty (1 - a^2, 2a; -(1 + a^2)) = \frac{r_+}{d-3} (\cos \alpha, \sin \alpha; -1).
 \tag{3.9}$$

k_0 is determined by the length of the vector (3.9),

$$p_L^2 = p_R^2 = \frac{1}{4} R_\infty^2 (1 + a^2)^2 = \left(\frac{r_+}{d-3} \right)^2,
 \tag{3.10}$$

and the mass-shell condition for the tachyon,

$$k_0^2 = \left(\frac{r_+}{d-3} \right)^2 - 1 = m_\infty^2 . \tag{3.11}$$

When the horizon size in string units is large, the asymptotic mass of the tachyon m_∞ is large as well, and the tachyon condensate (3.8) goes rapidly to zero at large distances. As the horizon shrinks, the tachyon becomes lighter and its condensate (3.8) spreads to larger and larger r . At

$$r_+ = d - 3 , \tag{3.12}$$

the tachyon becomes massless, and the condensate ceases to be normalizable. As in the two dimensional case, at that point we expect the system to make a transition to a string phase.

The entropy of perturbative single string states with mass m and charge q under a Kaluza-Klein gauge field $U(1)_L$ is given by the $Q \rightarrow 0$ limit of (2.40),

$$S_f(m, q) = 2\pi \left(m + \sqrt{m^2 - q^2} \right) . \tag{3.13}$$

The corresponding inverse temperature (2.29) is

$$\beta_f = \frac{4\pi}{1 - a^2} . \tag{3.14}$$

The chemical potential a is again the same as that in (3.5). Taking the limit $Q \rightarrow 0$ in the discussion of subsection 2.3 one finds that $1/\beta_f$ is the limiting temperature for fundamental strings with chemical potential a .

As before, the stretched horizon of the black hole is the region in which the local Hawking temperature exceeds the limiting temperature T_f , or $\beta_{bh}\sqrt{f(r)} < \beta_f$:

$$\frac{4\pi}{1 - a^2} \frac{r_+}{(d-3)} \sqrt{f(r)} < \frac{4\pi}{1 - a^2} . \tag{3.15}$$

For large black holes, $r_+ \gg 1$, one finds that the stretched horizon has proper size $\delta R \simeq 2/(1 - a^2)$, which is very similar to the two dimensional result given after eq. (2.46). As r_+ decreases, the size of the stretched horizon increases. It diverges at the transition point (3.12), where T_{bh} (3.5) reaches the limiting temperature T_f . In fact, the mass of the winding tachyon m_∞ (3.11), and temperatures T_{bh} , T_f , satisfy again the relation (2.47) for all dimensions d , masses m and charges q .

Following the logic of [1] and the discussion above, one expects that at the transition point (3.12) the black hole and string entropies should agree. Using equations (3.3), (3.4) and (3.13) we find that in general

$$\frac{S_{bh}}{S_f} = \frac{r_+}{d-2} . \tag{3.16}$$

In particular, at the transition point (3.12), we find that

$$\frac{S_{bh}}{S_f} = \frac{d-3}{d-2} , \tag{3.17}$$

independently of the mass and charge. The black hole and string entropies are not equal, but the black hole calculation was done using the leading order RN solution (3.1), which is expected to receive α' corrections. One might hope that these corrections will shift the ratio (3.17) to one.

It is interesting that the relation between the charge to mass ratio and the chemical potential given in the first line of (3.5) is the same for all systems considered in this paper: two dimensional and RN black holes, as well as fundamental strings in linear dilaton and flat spacetime. If the entropy depends on the mass and charge only via the combination $m + \sqrt{m^2 - q^2}$,

$$S_{bh}(m, q) = F(m + \sqrt{m^2 - q^2}) , \tag{3.18}$$

the relation on the first line of (3.5) follows for all F 's. Equation (3.18) is certainly valid in the large mass limit $m, q \rightarrow \infty$. We expect it to be a property of the full, α' corrected, black hole solution, but have not proved that this is indeed the case. ²

4. Discussion

In this section we discuss two aspects of the results of [1] and this paper. One is the question of g_s corrections to the properties of fundamental strings near the string/black hole transition point. The other is what happens in the extremal limit $m = q$.

One of the main results of [1] and this paper is that the black hole entropy should approach that of free fundamental strings when the Hawking temperature goes to that of fundamental strings with the same quantum numbers. One may object that the states we are matching have energies of order $1/g_s^2 \simeq 1/G_d$, at which the fundamental strings are no longer free. Indeed, as discussed in [20, 21], the effects of self gravity on individual string states are in general large near the string/black hole transition.

It is argued in [1] and this paper that the classical black hole sigma model describes an object that resembles more and more a cloud of free strings when the Hawking temperature goes to its limiting value, T_f . Therefore, its thermodynamics should approach that of free strings. The question of large g_s corrections to properties of fundamental strings near the transition translates in the black hole sigma model to the question whether that model has large string loop corrections for arbitrarily small g_s at $T_{bh} \simeq T_f$.

Near the transition, one expects this sigma model to receive large α' corrections but the g_s corrections are, at least formally, arbitrarily small. If the string loop corrections to the classical black hole background are indeed small, one would conclude that the same is true for the thermal ensemble of free strings at the temperature and chemical potential discussed above. Conversely, if the free string ensemble receives large corrections near the transition point, then the black hole sigma model must receive large quantum corrections there for arbitrarily small g_s . In any case, it seems that the matching between the classical black hole background and free strings must hold.

²If (3.18) is invalid, the chemical potential a is not a function of the charge to mass ratio, but depends on both variables.

In the discussion of charged black holes in sections 2 and 3 we focused on the non-extremal case. It is natural to ask what happens when we take the extremal limit $q \rightarrow m$. In this limit the black hole temperature T_{bh} and the limiting fundamental string temperature T_f go to zero (see (2.26), (2.41), (3.5), (3.14)). Since the two temperatures are equal, one might expect the black hole and string entropies to agree, and the wound tachyon in the euclidean geometry to be massless.

These expectations are in apparent disagreement with the facts. In the two dimensional case, the mass of the tachyon at infinity is given by (2.39). It is independent of the charge to mass ratio of the black hole, and is large for small Q . Also, the black hole and string entropies (2.30) and (2.40) do not agree for $q = m$, except at $Q^2 = 2$, which is the transition point for all q . Similarly, for d dimensional RN black holes, the mass of the tachyon at infinity (3.11) is in general non-zero for $r_- = r_+$, and the black hole and string entropies (3.4) and (3.13) are in general different.

To see how this conundrum is resolved in string theory, consider the two dimensional case of section 2, which has the advantage of having an exact coset description. This description gives the geometry (2.8)–(2.10). For $a \neq 1$, this geometry is the same as that of (2.1)–(2.3). For $a = 1$, it changes qualitatively. In particular, the asymptotic behavior changes from linear dilaton to AdS_2 (see (2.14)). This AdS_2 can be thought of as the near-horizon geometry of the extremal black hole (2.1).

Thus, while for $a \neq 1$, the CFT (2.7) describes the full black hole geometry, for $a = 1$ it describes only the near-horizon region. This resolves the above puzzles, essentially by avoiding them. The linear dilaton region (2.6) is pushed to infinity and is no longer part of the space (2.14). Therefore, the behavior of the tachyon condensate and the disagreement between the black hole and string entropies (2.30), (2.40) for generic Q are unimportant since they refer to a region that is not part of the geometry.

One can ask whether it is possible to compute the entropy of extremal black holes, which is given by (2.30) with $q = m$, using the coset geometry (2.14). We expect that it should be possible to do that by mapping string theory on the AdS_2 coset to a dual CFT using the AdS/CFT correspondence.

In the higher dimensional RN geometries we do not have the analog of the coset description of the exact background, but we expect the situation to be similar. For $q < m$ we expect there to be an exact worldsheet CFT that describes the α' corrected RN geometry. For $q = m$ such a geometry should not exist; instead, one should be able to construct a worldsheet CFT which describes the near-horizon geometry of the extremal black hole, $AdS_2 \times S^{d-2}$, from which one would be able to calculate the black hole entropy (3.4).

Acknowledgments

We thank O. Lunin for discussions. This work is supported in part by the BSF — American-Israel Bi-National Science Foundation. A.G. is also supported in part by the Israel Science Foundation and the EU grant MRTN-CT-2004-512194. A.G. thanks the Theoretical Physics Division at CERN and the EFI at the University of Chicago for hospitality. The work of D.K. is also supported in part by DOE grant DE-FG02-90ER40560.

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